QUESTION 1 (Start a new page)

Marks

a) Express 205° as an exact radian value. 1

b) A sector of a circle of radius 8cm has an angle of 120°. Calculate the arc length of the sector to 3 significant figures. 2

c) Find the first derivative of:

(i) $f(x) = (5x^2 - 1)^4$ 2

(ii) $g(x) = \dfrac{3x+1}{2x-1}$ 2

(iii) $y = \ln(4x+5)$ 1

d) Fifty identical cards numbered 1 to 50 are placed in a hat, and one is drawn at random. Find the probability that this card will be either less than 30 or divisible by 5. 2

e) Factorise fully: $16x^3 - 64x$. 2

QUESTION 2 (Start a new page)

Marks

a) Given that for the function $y = f(x)$, $\frac{d^2y}{dx^2} = 6x - 2$ and that there exists a stationary point at $(1,2)$, find:

(i) The equation of the function. **3**

(ii) The coordinates of the point of inflection. **2**

b) Evaluate $\lim_{x \to 3} \frac{3 + 2x - x^2}{x - 3}$. **2**

c) Given the function $y = -2\sin 3x$.

(i) State the period of the function. **1**

(ii) Sketch the graph of $y = -2\sin 3x$ for $0 \le x \le \pi$. **2**

d) Simplify: $\frac{3^n \times 9^{n-1}}{27^{2n}}$. **2**

QUESTION 3 (Start a new page)

Marks

a) A parabola has as its equation $8y = x^2 - 4x - 28$

(i) Write this equation in the form $(x-h)^2 = 4a(y-k)$ and hence or otherwise give the coordinates of the vertex. 3

(ii) Find the coordinates of the focus. 1

b) If the quadratic equation $2x^2 - 3x + 4 = 0$ has roots α and β, find:

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

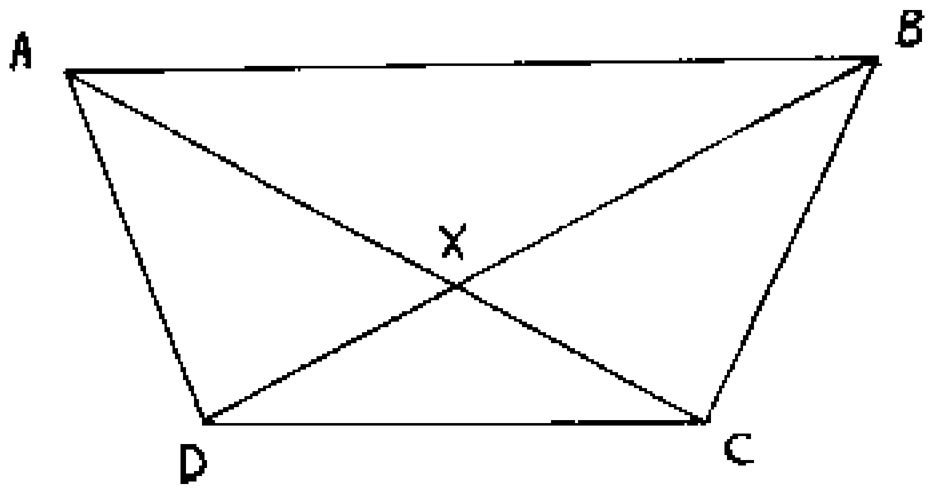
(iii) $\alpha\beta^2 + \beta\alpha^2$ 1

c) ABCD is a trapezium in which AB is parallel to DC. The diagonals intersect at X. AB = 12 cm. DC = 8 cm and AC = 10 cm.

Copy the diagram onto your answer page and clearly label it with the above information.

(i) Prove that ΔAXB is similar to ΔCXD. 3

(ii) Hence, find the length of AX. 2



Not to scale

QUESTION 4 (Start a new page)

Marks

a) A and B are points on the number plane with coordinates (6, 0) and (0, –8) respectively. A general point P(x, y) is moving such that PA is perpendicular to PB. 3

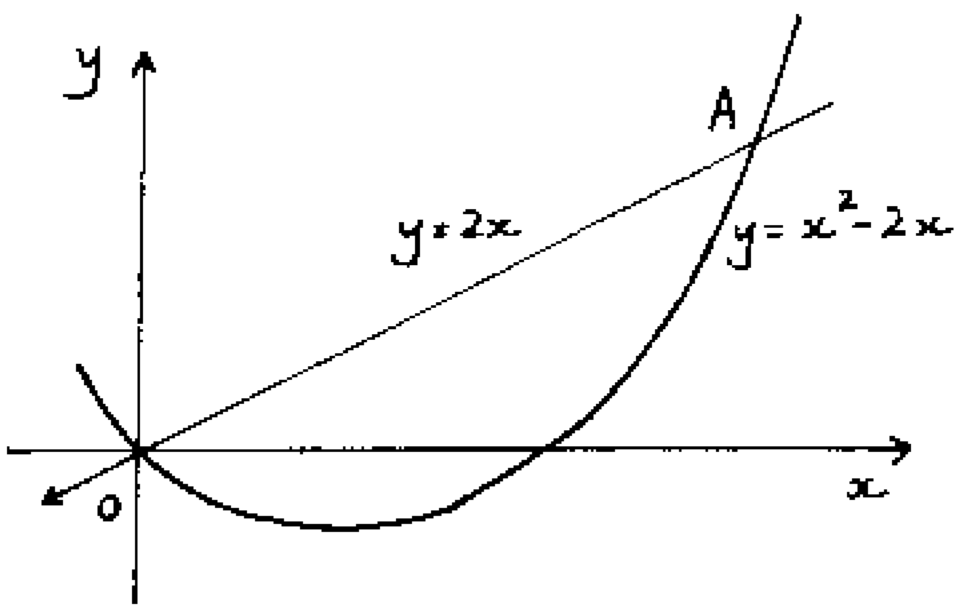
Show that the locus of P has equation $x^2 + y^2 - 6x + 8y = 0$.

b) If $y = ln\left(\frac{1-x}{1+x}\right)$ show that $\frac{dy}{dx} = \frac{-2}{1-x^2}$. 3

Hence or otherwise, evaluate $\int_0^{\frac{1}{2}} \frac{dx}{1-x^2}$.

c) Find $\int \frac{t^2 - 2t}{t^3} dt$. 2

d)



Not to scale

The graphs of $y = 2x$ and $y = x^2 - 2x$ are shown in the diagram. They intersect at points 0 and A.

(i) Find the coordinates of point A. 1

(ii) Find the area enclosed by these two graphs. 3

QUESTION 5 (Start a new page)

Marks

a) The first four terms of a series are 3, x, y, 192. Find the real values of x and y if the series is:

(i) arithmetic. 2

(ii) geometric. 2

b) A weight lifter in training tires with each lift such that he can only lift 90% of the preceding lift. If his first lift was 200kg:

(i) What weight will he raise on his fifth lift? 2

(ii) Theoretically, what would be the total of the weights lifted by the time he was totally exhausted? 2

c) Evaluate $\sum_{n=3}^{50}(100-4n)$. 2

d) If $\cos\beta = \frac{3}{7}$ and $\sin\beta < 0$, find the exact value of $\tan\beta$. 2

QUESTION 6 (Start a new page)

Marks

a) Consider the curve given by $y = 3x^2 - x^3 + 9x - 2$ 6

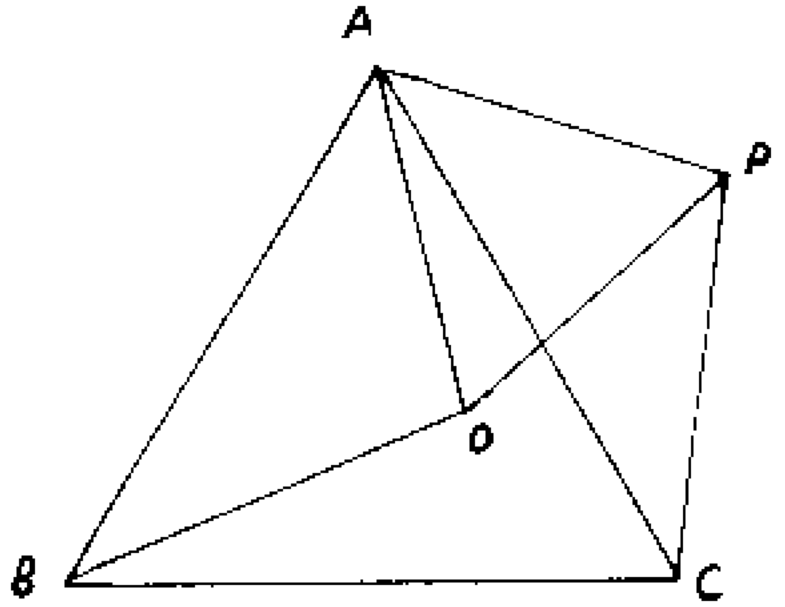
(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(ii) Find the coordinates of the stationary points and determine their nature.

(iii) Sketch the graph of the function for the domain $-2 \le x \le 5$.

(iv) State the minimum value of the function over this domain.

b) 6



Not to scale

In the figure triangles ACB and APO are equilateral.

Copy this diagram onto your answer sheet and include all the given information.

(i) Explain why $\angle BAO = \angle PAC$.

(ii) Prove $\Delta AOB \equiv \Delta APC$.

(iii) Hence, prove $OB = CP$.

QUESTION 7 (Start a new page)

Marks

a) The depth of water in the cross–section of a creek was measured and recorded in the table below.

DISTANCE FROM BANK(m)	0	1	2	3	4
DEPTH(m)	0.4	0.8	1.5	1.3	0.3

2

Using these five function values, apply Simpson's Rule to find the cross–sectional area of the creek (to 2 decimal places).

b) A sprinter knows the probability of equalling or bettering their personal best time in any race is 0.2. By drawing a tree diagram or otherwise find the probability that:

(i) A personal best will <u>not</u> be achieved in 3 successive races. 2

(ii) A personal best will be achieved at least once in 3 successive races. 1

(iii) A personal best will be achieved in 2 out of 3 successive races. 2

c) Over what domain is the function $y = \sqrt{x^2 - 2x - 8}$ defined? 3

d) Show that $x^2 + kx + k - 1 = 0$ has real roots for all values of k. 2

QUESTION 8 (Start a new page)

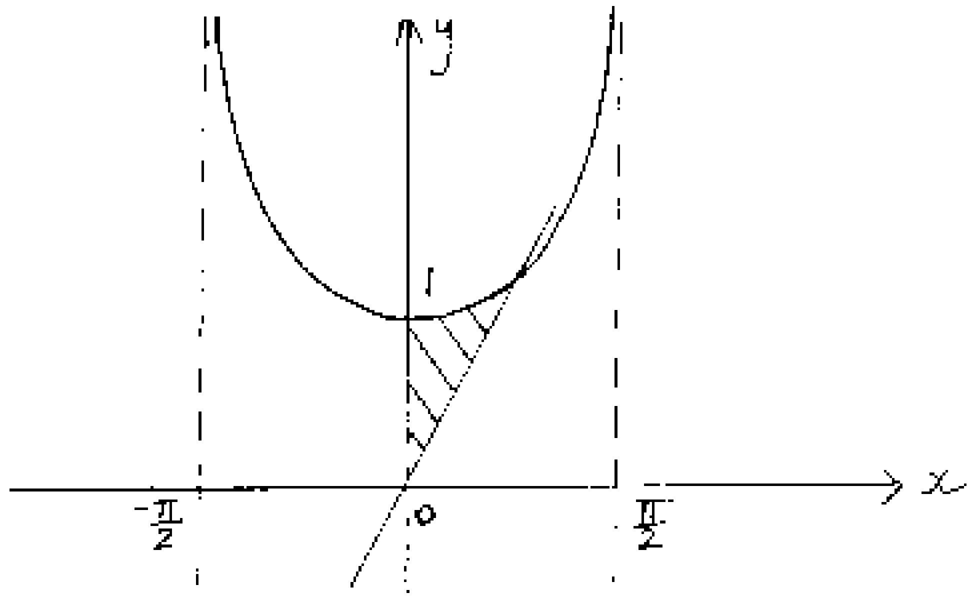
Marks

a) An employee joins a superannuation fund on their eighteenth birthday by investing \$3000 in the fund. This investment is repeated every six months until retirement on their sixtieth birthday. (Last payment is made six months before retirement).

If the interest rate of 12.25% per annum is compounded six monthly, calculate the amount they will receive on retirement (to nearest dollar). 2

b) (i) The diagram shows part of the graphs of both $y = \sec x$ and $y = \frac{4\sqrt{2}}{\pi}x$. 2

Show that $\left(\frac{\pi}{4}, \sqrt{2}\right)$, satisfies both of these equations.



(ii) The area bounded by the curve $y = \sec x$, $y = \frac{4\sqrt{2}}{\pi}x$ and the y–axis is rotated about the x–axis. Find the volume of the solid of revolution. 3

c) A block of ice is removed from the refrigerator. The rate at which the ice melts is proportional to the amount remaining, ie. $\frac{dM}{dt} = -kM$, where M is measured in grams and time in minutes.

(i) Show that $M = M_0 e^{-kt}$ is a solution of the equation $\frac{dM}{dt} = -kM$. 1

(ii) After 35 minutes, only half of the ice remains. Find the value of k to 3 significant figures. 2

(iii) If, at a certain time, only 5% of the block remains, how long would it have been since the block was removed from the refrigerator (answer in hours and minutes)? 2

QUESTION 9 (Start a new page)

Marks

a) A closed cylindrical can of radius r cm and height h cm is to be made from a sheet of metal with area $300\pi\ cm^2$. There is 10% wastage of the sheet in manufacturing the can. 6

(i) Show that $h = \dfrac{135 - r^2}{r}$.

(ii) Find an expression for the volume V as a function of r.

(iii) Find the value of r which gives the maximum volume.

(iv) Calculate the maximum volume.

b) A colony of bacteria is being studied. After t seconds the rate at which the population is changing is given by $R = 1000 + \dfrac{500}{1+t}$, where R measures the change in the number of bacteria per second.

(i) What is the rate of change in the number of bacteria in the colony at the beginning of the experiment. 1

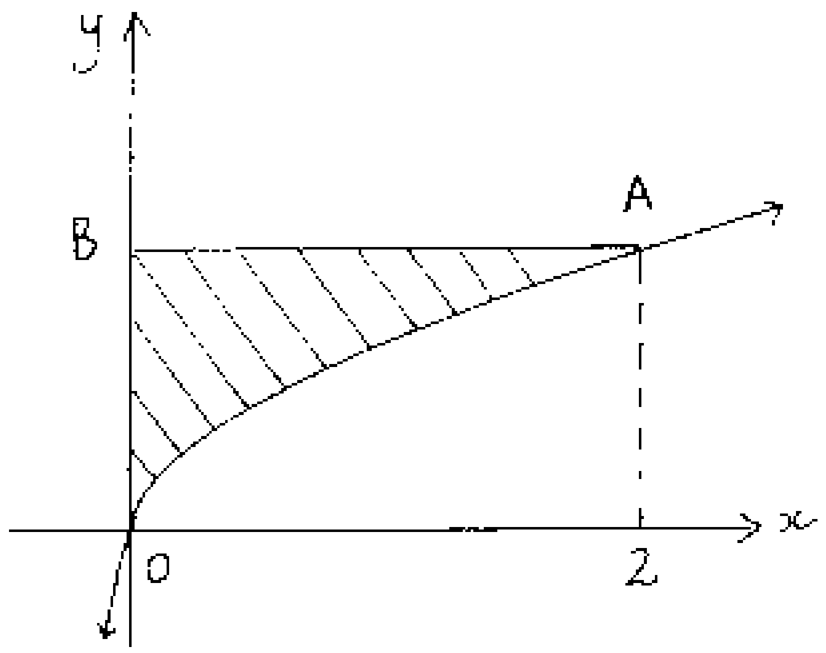
(ii) Sketch the graph of R against t. 2

(iii) In the first 9 seconds, what will be the total increase in the number of bacteria (answer to nearest hundred). 3

QUESTION 10 (Start a new page)

Marks

a) The diagram shows the graph of $y = \log_e(x+1)$.



Not to scale

(i) Find the exact coordinate of the point A. **1**

(ii) Evaluate $\int_0^2 \log_e(x+1)\,dx$. **3**

(iii) A volume of solid of revolution is found by rotating this area about the y–axis. Calculate this volume. **3**

b) A particle moves so that its displacement is given by $x = te^{-t}$.

(i) Show that $v = e^{-t}(1-t)$ **1**

(ii) Find when the particle is at rest. **1**

(iii) Find the acceleration as a function of t. **1**

(iv) Describe the motion of the particle for $t \geq 0$. **2**

QUESTION 1

a) $205 \times \frac{\pi}{180} = \frac{41\pi}{36}$

b) $120° = \frac{2\pi}{3}$

$l = r\theta$

$= 8 \times \frac{2\pi}{3} = \frac{16\pi}{3} = 16 \cdot 8$ (3sf)

c) (i) $f'(x) = 4(5x^2-1)^3 \cdot 10x$

$= 40x(5x^2-1)^3$

(ii) $g'(x) = \frac{3(2x-1) - 2(3x+1)}{(2x-1)^2}$

$= \frac{6x - 3 - 6x - 2}{(2x-1)^2}$

$= \frac{-5}{(2x-1)^2}$

(iii) $\frac{dy}{dx} = \frac{4}{4x+5}$

d) $P(<30 \text{ or } \div 5) = P(<30) + P(\div 5) - P(<30 \,\&\, \div 5)$

$= \frac{29}{50} + \frac{10}{50} - \frac{5}{50}$

$= \frac{34}{50}$

e) $16x^3 - 64x = 16x(x^2-4)$

$= 16x(x-2)(x+2)$

QUESTION 2

a) (i) $\frac{d^2y}{dx^2} = 6x - 2$

$\frac{dy}{dx} = 3x^2 - 2x + c$

at $x = 1$, $\left(\frac{dy}{dx} = 0\right)$: $3 - 2 + c = 0$

$c = -1$

$\frac{dy}{dx} = 3x^2 - 2x - 1$

$y = x^3 - x^2 - x + k$

$(1, 2)$ satisfies this eqn

$2 = 1 - 1 - 1 + k$

$k = 3$

$\therefore y = x^3 - x^2 - x + 3$

(ii) For pt of inflection,

$\frac{d^2y}{dx^2} = 0$ & a change in concavity exist.

ie $6x - 2 = 0$

$x = \frac{1}{3}$

for $x < \frac{1}{3}$, say $x = 0$ $\frac{d^2y}{dx^2} = -2 < 0$

for $x > \frac{1}{3}$, say $x = 1$ $\frac{d^2y}{dx^2} = 4 > 0$

Since concavity changes from down to up about $x = \frac{1}{3}$, then $\left(\frac{1}{3}, 2\frac{16}{27}\right)$ is a pt. of inflection

b) $\lim_{x \to 3} \frac{3 + 2x - x^2}{x - 3}$

$= \lim_{x \to 3} \frac{-(x-3)(x+1)}{x-3}$

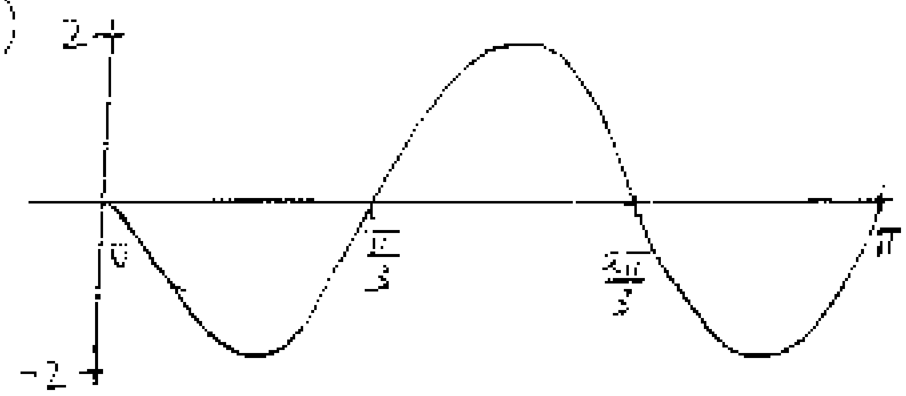
$= \lim_{x \to 3} -(x+1)$

$= -4$

c) $y = -2\sin 3x$

(i) Period $= \frac{2\pi}{3}$

(ii)



d) $\frac{3^m \times 9^{m+1}}{27^{2m}} = \frac{3^m \times 3^{2m+2}}{3^{6m}}$

$= 3^{2-3m}$

Question 3

a) $8y = x^2 - 4x - 28$

$4 + 8y + 28 = x^2 - 4x + 4$

$8y + 32 = (x-2)^2$

$\therefore (x-2)^2 = 4 \times 2(y+4)$

(i) vertex $= (2, -4)$

(ii) focal length $= 2$

focus $= (2, -2)$

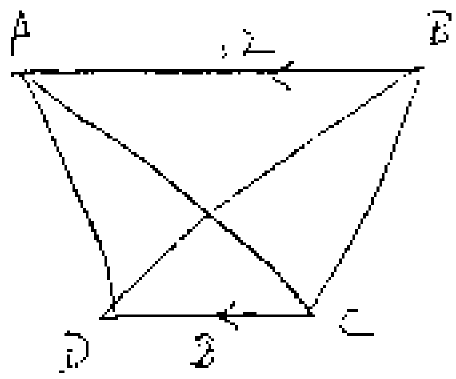
b) $2x^2 - 3x + 4 = 0$

(i) $\alpha + \beta = \frac{-(-3)}{2} = \frac{3}{2}$

(ii) $\alpha\beta = \frac{4}{2} = 2$

(iii) $\alpha\beta(\beta + \alpha) = 2 \times \frac{3}{2} = 3$

c)



(i) In ΔAXB & CXD:

$\angle AXB = \angle CXD$ (vertically opp. ∠)

$\angle BAX = \angle DCX$ (alt. angles; AB ∥ CD)

$\therefore \Delta AXB ||| \Delta CXD$ (equiangular)

(ii) Let $AX = y$

$XC = 10 - y$

$\frac{AB}{CD} = \frac{AX}{XC}$

$\frac{12}{8} = \frac{y}{10-y}$

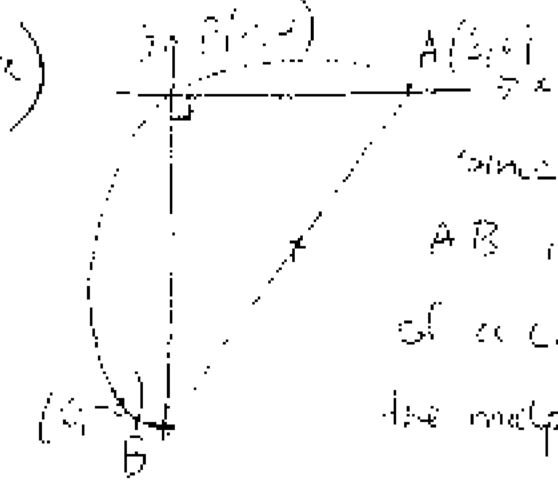
$3(10-y) = 2y$

$30 - 3y = 2y$

$y = 6$

Question 4

a)



Since $PA \perp PB$, then AB is the diameter of a circle with centre the midpoint AB

midpoint AB $= (3, -4)$

Radius $= 5$

Eqⁿ of locus $(x-3)^2 + (y+4)^2 = 5^2$

$x^2 - 6x + 9 + y^2 + 8y + 16 = 25$

$x^2 + y^2 - 6x + 8y = 0$

b) $y = \ln\left(\frac{1-x}{1+x}\right)$

$= \ln(1-x) - \ln(1+x)$

$\frac{dy}{dx} = \frac{-1}{1-x} - \frac{1}{1+x}$

$= \frac{-(1+x)-(1-x)}{(1+x)(1-x)}$

$= \frac{-2}{1-x^2}$

$\int_0^{\frac{1}{2}} \frac{dx}{1-x^2} = -\frac{1}{2}\left[\ln\left(\frac{1-x}{1+x}\right)\right]_0^{\frac{1}{2}}$

$= -\frac{1}{2}\left(\ln\frac{1}{3} - \ln 1\right)$

$= -\frac{1}{2}\ln\frac{1}{3}$ OR $\frac{1}{2}\ln 3$

c) $\int \frac{t^2-2t}{t^3}\, dt$

$= \int \frac{t-2}{t^2}\, dt$

$= \int \left(\frac{1}{t} - 2t^{-2}\right) dt$

$= \ln t + \frac{2}{t} + c$

d) (i) $y = 2x$
$y = x^2 - 2x$

$2x = x^2 - 2x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ or $x = 4$

$\therefore A = (4, 8)$

(ii) Area $= \int_0^4 \left[2x - (x^2 - 2x)\right] dx$

$= \int_0^4 (4x - x^2)\, dx$

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \left(2\times 4^2 - \frac{4^3}{3}\right) - 0$

$= 10\frac{2}{3}\, u^2$

QUESTION 5

a) (i) $d = x - 3 = y - x = 192 - y$

Now $x - 3 = y - x$

$2x - 3 = y$ ---- (1)

also $y - x = 192 - y$

$2y = 192 + x$ --- (2)

Solving (1) & (2).

$2(2x-3) = 192 + x$

$4x - 6 = 192 + x$

$x = 66$

$\therefore y = 2\times 66 - 3 = 129$

A.P. 3, 66, 129, 192

(ii) $r = \frac{x}{3} = \frac{y}{x} = \frac{192}{y}$

Now $\frac{x}{3} = \frac{y}{x}$

$x^2 = 3y$

$y = \frac{x^2}{3}$ ---- (1)

Also $\frac{y}{x} = \frac{192}{y}$

$y^2 = 192x$ ---- (2)

Solving (1) & (2):

$\frac{x^4}{9} = 192x$

$x^4 = 1728x$

$x^4 - 1728x = 0$

$x(x^3 - 1728) = 0$

$x = 0$ or $x = 12$

Since $x \neq 0$ $x = 12$ only.

$\therefore y = \frac{12^2}{3} = 48$

$\therefore$ G.P. 3, 12, 48, 192

b) (i) GP, $a = 200$, $r = 0.9$, $T_5 = ?$

$T_n = ar^{n-1}$

$T_5 = 200 \times 0.9^4$

$= \underline{131.22\text{ kg}}$

(ii) $S = \frac{a}{1-r} = \frac{200}{1-0.9} = \underline{2000\text{ kg}}$

c) $a = T_3 = 100 - 4 \times 3 = 88$

$T_{50} = 100 - 4 \times 50 = -100$

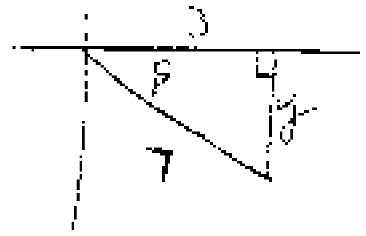
$n = 50 - 3 + 1 = 48$

Series is an AP

$S_{48} = \frac{48}{2}\left(88 + (-100)\right)$

$= \underline{-288}$

d) $\cos\beta > 0$ and $\sin\beta < 0$ so β lies in 4th quadrant



By Pythagoras,

$y = \sqrt{40} = 2\sqrt{10}$

$\therefore \tan\beta = -\frac{2\sqrt{10}}{3}$

Question 6

i) (i) $y = 3x^2 - x^3 + 9x - 2$

$\frac{dy}{dx} = 6x - 3x^2 + 9$

$\frac{d^2y}{dx^2} = 6 - 6x$

(ii) For stat. pts $\left(\frac{dy}{dx} = 0\right)$

i.e. $6x - 3x^2 + 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or -1

at $x = 3$, $\frac{d^2y}{dx^2} = -12 < 0$ ∴ local max at $(3, 25)$

at $x = -1$, $\frac{d^2y}{dx^2} = 12 > 0$ ∴ local min at $(-1, -7)$

For possible pts of inflection $\left(\frac{d^2y}{dx^2} = 0\right)$

i.e. $6 - 6x = 0$

$x = 1$

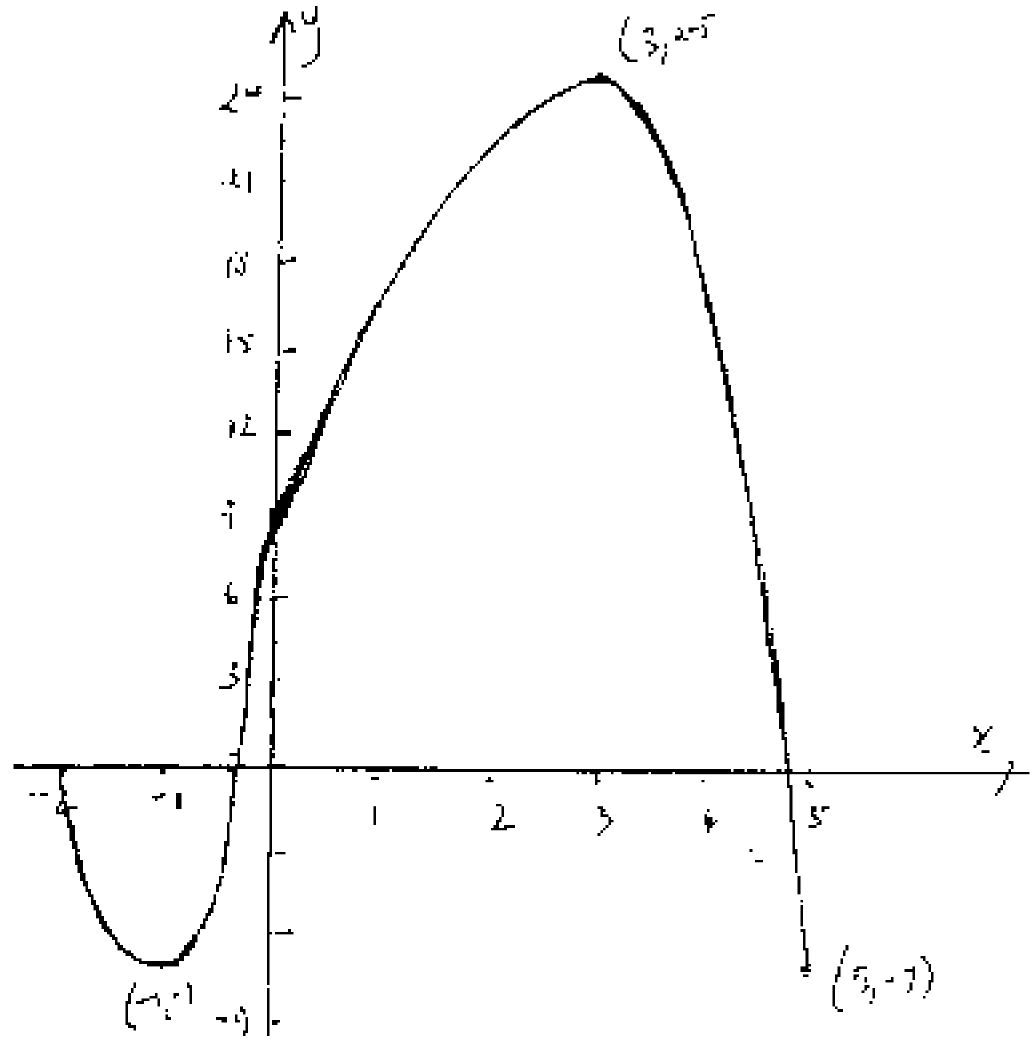
for $x < 1$, say $x = -1$, $\frac{d^2y}{dx^2} = 12 > 0$

for $x > 1$, say $x = 3$, $\frac{d^2y}{dx^2} = -12 < 0$

since concavity changes from up to down about $x = 1$, then $(1, 9)$ is a pt of inflection.

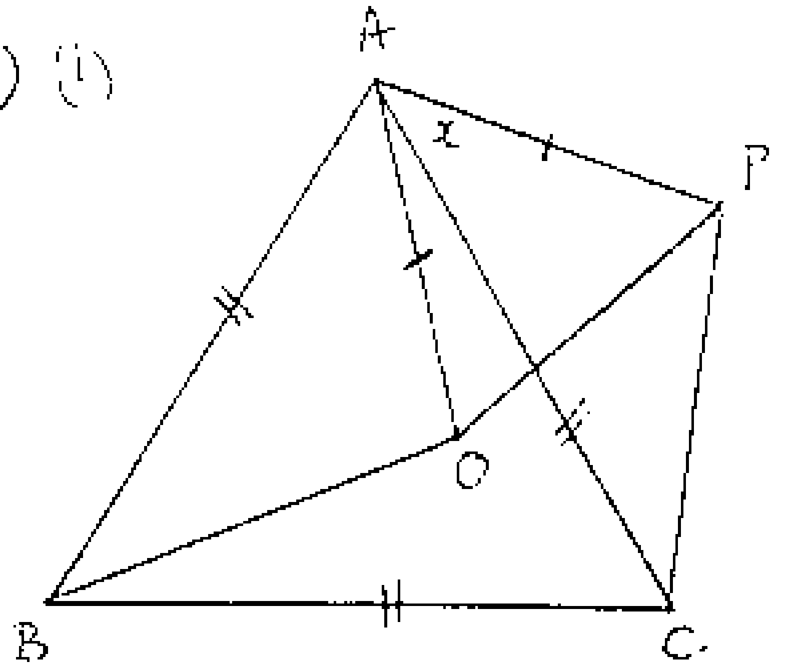
(iii) at $x = -2$, $y = 0$

at $x = 5$, $y = -7$



(iv) Minimum value is -7

b) (i)



(ii) Let $\angle PAC = x$

$\angle CAO = 60 - x$ (ΔAPO is equilateral; $\angle PAO = 60$)

$\angle BAC = 60^\circ$ (ΔABC is equilateral, $\angle BAC = 60$)

$\angle BAO = 60 - (60 - x)$

$= x$

$\therefore \angle PAC = \angle BAO \quad (= x)$

(iii) In ΔAOB & APC:

$AB = AC$ (ΔABC is equilateral)

$AO = AP$ (ΔACP is equilateral)

$\angle BAO = \angle PAC$ (above)

$\Delta AOB \equiv \Delta APC$ (SAS)

(iv) $OB = PC$ (corresponding sides of congruent Δs)

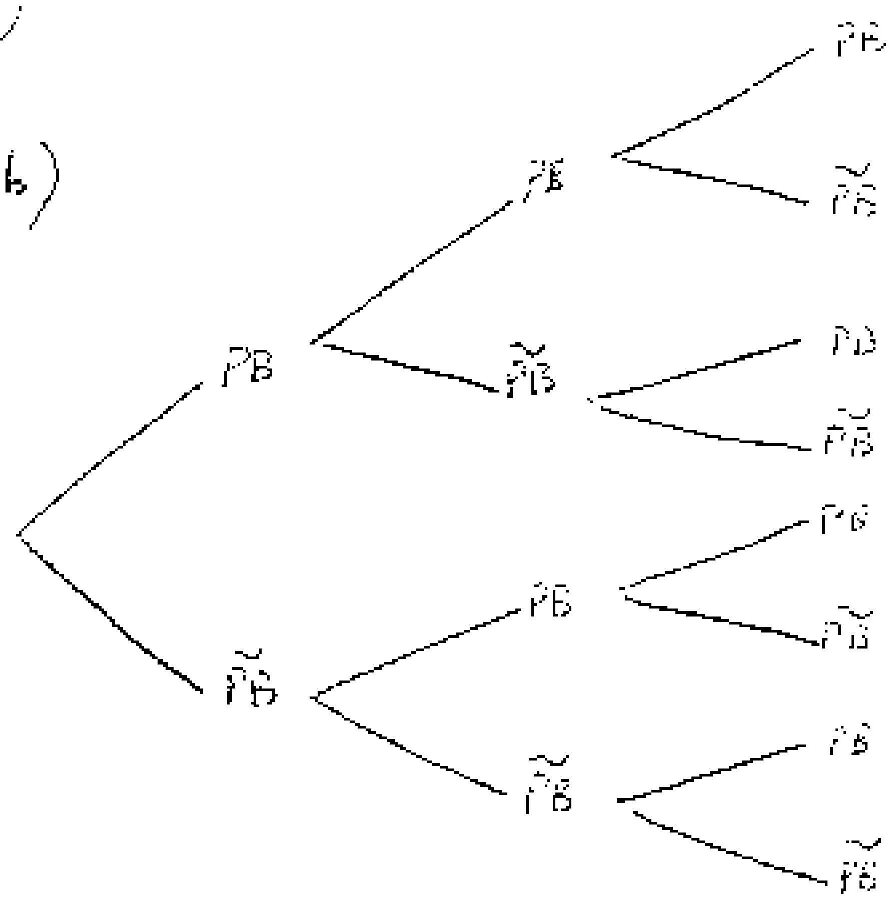
QUESTION 7

a) $A = \frac{2-0}{6}\left[0.4 + 4\times 0.3 + 1.5\right] + \frac{4-2}{6}\left[1.5 + 4\times 1.3 + 0.3\right]$

$= 4.0\dot{3}$

$= 4.03$ (2 dp)

b)



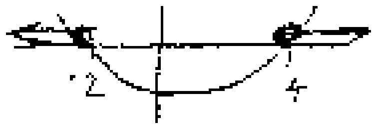
(i) $(0.8)^3 = 0.512$

(ii) $1 - (0.8)^3 = 0.488$

(iii) $3 \times (0.2)^2(0.8) = 0.096$

c) $x^2 - 2x - 8 \geq 0$

$(x-4)(x+2) \geq 0$



$x \leq -2$ or $x \geq 4$

d) For real roots, $\Delta \geq 0$

$\Delta = k^2 - 4.1.(k-1)$

$= k^2 - 4k + 4$

$= (k-2)^2$

≥ 0 for all values of k.

Question 3

a) $n = 54 - 18 + 1 = 42$ years $= 42 \times 2 + 1$

$= 85$ payments

12.25% pa $= 6.125\%$ p/6 months

First \$3000 amounts to $3000(1.06125)^{85}$

2nd " " " $3000(1.06125)^{84}$

⋮

last " " " $3000(1.06125)^{1}$

Total value of investment

$3000\left[1.06125 + \cdots + 1.06125^{84} + 1.06125^{85}\right]$

$= 3000\left[\dfrac{1.06125^{85} - 1}{0.06125}\right] \times 1.06125$

$= \$8,052,647$ (nearest dollar)

b)(i) Sub $\left(\frac{\pi}{4}, \sqrt{2}\right)$ into each equation: $\sec\frac{\pi}{4} = \sqrt{2}$ ∴ true

$\frac{4\sqrt{2}}{\pi} \times \frac{\pi}{4} = \sqrt{2}$ ∴ true

(ii) $V = \pi\int_0^{\frac{\pi}{4}} \sec^2 x - \left[\frac{4\sqrt{2}x}{\pi}\right]^2 dx$

$= \pi\int_0^{\frac{\pi}{4}} \left(\sec^2 x - \frac{32}{\pi^2}x^2\right) dx$

$= \pi\left[\tan x - \frac{32}{3\pi^2}x^3\right]_0^{\frac{\pi}{4}}$

$= \pi\left[\left(\tan\frac{\pi}{4} - \frac{32}{3\pi^2}\cdot\frac{\pi^3}{64}\right) - 0\right]$

$= \pi\left[1 - \frac{\pi}{6}\right] u^3$

c)

(i) $M = M_0 e^{-kt}$

$\frac{dM}{dt} = -k M_0 e^{-kt}$

$= -kM$ as required.

(ii) $\frac{M_0}{2} = M_0 e^{-k \times 35}$

$\frac{1}{2} = e^{-35k}$

$\ln\frac{1}{2} = -35k$

$k = \frac{\ln\frac{1}{2}}{-35}$

$k \doteqdot 0.0198$ (3 s.f)

(iii) $\frac{1}{20} = e^{-0.0198t}$

$\ln\frac{1}{20} = -0.0198t$

$t = \frac{\ln\frac{1}{20}}{-0.0198}$

$t \doteqdot 151$ min OR 2hrs 31 min

Question 4

(i) Area required $= 300\pi - 30\pi = 270\pi$

∴ $2\pi r^2 + 2\pi rh = 270\pi$

$2\pi r(r+h) = 270\pi$

$r + h = \frac{270\pi}{2\pi r}$

$h = \frac{135}{r} - r$

∴ $h = \frac{135 - r^2}{r}$

(ii) $V = \pi r^2 h$

$= \pi r^2\left(\frac{135 - r^2}{r}\right)$

∴ $V = 135\pi r - \pi r^3$

(iii) For max. volume $\left(\frac{dV}{dr} = 0\right)$

$\frac{dV}{dr} = 135\pi - 3\pi r^2$

ie $135\pi - 3\pi r^2 = 0$

$45 - r^2 = 0$

$r = \sqrt{45}$ $(r > 0)$

$\frac{d^2V}{dr^2} = -6\pi r$

for $r=\sqrt{45}$, $\frac{d^2V}{dr^2} = -6\pi\sqrt{45} < 0$

∴ concave down

∴ local max at $r=\sqrt{45}$

Since the function is continuous, then $r=\sqrt{45}$ is the absolute maximum.

(iv) When $r=\sqrt{45}$, $V = 135\pi\sqrt{45} - \sqrt{45}^3\pi$

$= 90\sqrt{45}\pi$

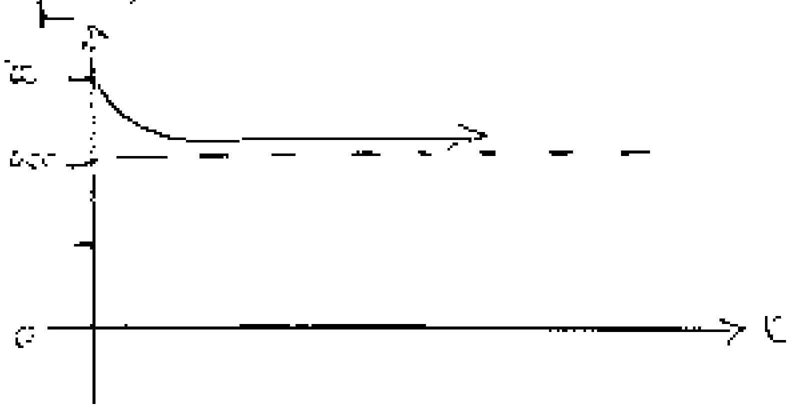
$= \underline{270\pi\sqrt{5}\ u^3}$

b) (i) $R = 1000 + \frac{500}{1+t}$

at $t=0$, $R = 1000 + 500 = \underline{1500}$

∴ initially bacteria decreasing at a rate of 1500 bacteria/second.

(ii) as $t \to \infty$, $R \to 1000$



(iii) Let $R = \frac{dP}{dt}$

$\therefore \frac{dP}{dt} = 1000 + \frac{500}{1+t}$

$P = \int_0^9 \left(1000 + \frac{500}{1+t}\right) dt$

$= \left[1000t + 500\ln(1+t)\right]_0^9$

$= 9000 + 500\ln 10$

$\therefore P = \underline{10200}$ (to nearest hundred)

Question 10

a) (i) $A = (2, \ln 3)$

(ii) $y = \log_e(x+1)$

$e^y = x+1$

$x = e^y - 1$

Shaded area $= \int_0^{\ln 3} (e^y - 1)\, dy$

$= \left[e^y - y\right]_0^{\ln 3}$

$= (e^{\ln 3} - \ln 3) - (e^0 - 0)$

$= 3 - \ln 3 - 1$

$= \underline{2 - \ln 3}$

Required Integral

$= 2 \times \ln 3 - (2 - \ln 3)$

$= \underline{3\ln 3 - 2}$

(iii) $V = \pi \int_0^{\ln 3} (e^y - 1)^2\, dy$

$= \pi \int_0^{\ln 3} (e^{2y} - 2e^y + 1)\, dy$

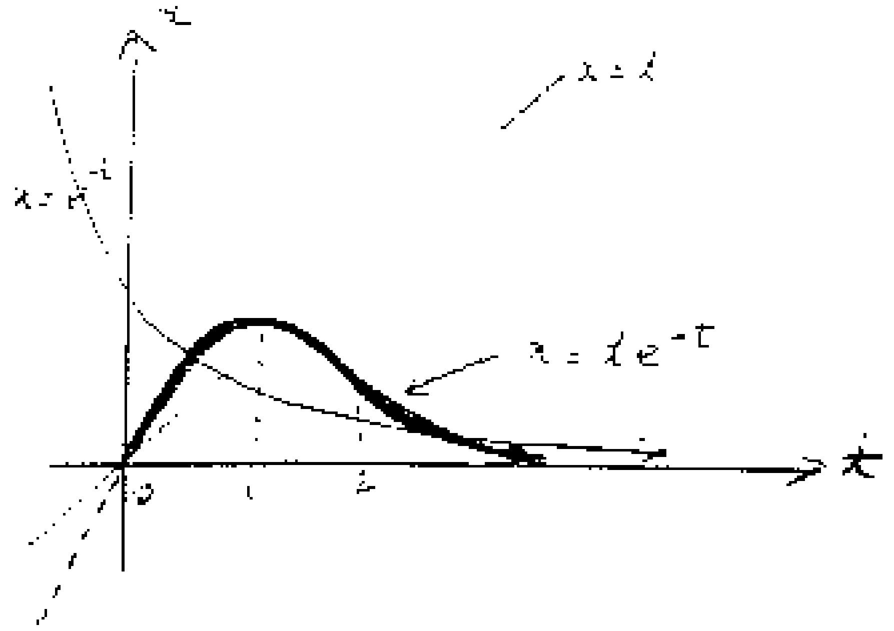
$= \pi \left[\frac{1}{2}e^{2y} - 2e^y + y\right]_0^{\ln 3}$

$= \pi \left\{\left(\frac{1}{2}e^{2\ln 3} - 2e^{\ln 3} + \ln 3\right) - \left(\frac{1}{2}e^0 - 2e^0\right)\right\}$

$= \pi \left\{\left(\frac{9}{2} - 6 + \ln 3\right) - \left(\frac{1}{2} - 2\right)\right\}$

$= \underline{\pi \ln 3}$

b) $x = te^{-t}$



(i) $v = t \times -e^{-t} + e^{-t} \times 1$

$= -te^{-t} + e^{-t}$

$\underline{v = e^{-t}(1-t)}$

(ii) Particle at rest when $v = 0$.

$e^{-t}(1-t) = 0$

$e^{-t} \neq 0 \qquad 1 - t = 0$

$\underline{t = 1}$ only.

(iii) $a = e^{-t} \times (-1) + (1-t) \times -e^{-t}$

$= -e^{-t} - e^{-t} + te^{-t}$

$\underline{a = e^{-t}(t-2)}$

(iv) As $t \to \infty$, $x \to 0$

$v \to 0$

$a \to 0$

$a = 0$ at $t = 2$

$a < 0$ for $t < 2$

$a > 0$ for $t > 2$

The particle starts at the origin and is moving right, slowing down as time is to left. It comes to rest at $t = 1$. It changes direction speeding up until $t = 2$ when it meets a positive force, causing it to slow down but never reaching the origin.